Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18-20 December 2011, Dhaka, Bangladesh

ICME11-TH-035

PRANDTL NUMBER EFFECT ON MIXED CONVECTION IN A VERTICAL CHANNEL WITH A HEAT-GENERATING OBSTACLE

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ABSTRACT
This paper reports the results of a numerical study of Prandtl number effect on free and forced convective flow and heat transfer in an octagonal vertical channel. A horizontal heat-generating hollow circular obstacle is placed at the centre of the octagon. Equations for the conservation of mass, momentum and energy are solved by employing the finite element simulation. The input and output opening are situated at the bottom and top surface respectively. The vertical and the inclined walls of the octagon are perfectly insulated. Velocity and temperature fields in terms of streamlines and isotherms for different combinations of governing parameters namely, Prandtl number (*Pr*) and Richardson number (*Ri*) are displayed graphically. Finally, average Nusselt number (*Nu*) and maximum temperature (θ_{max}) of fluid are depicted also. The highest *Nu* and the lowest θ_{max} are found for the maximum considered value of *Pr* in all convection regimes.

Keywords: Heat-Generation, Vertical Channel, Mixed Convection And Finite Element Method.

1. INTRODUCTION
Mixed convection in channels is of great interest of the phenomenon in many technological processes, such as the design of solar collectors, thermal design of buildings, air conditioning and the cooling of electronic circuit boards. Alteration of heat transfer in channels due to introduction of obstacles, partitions and fins attached to the wall(s) has received sustained attention. Many authors have considered mixed convection in vented enclosures with obstacles, partitions and fins, thereby altering the convection flow phenomenon. Real cavities in practice are often found to have different shapes rather than rectangular one. A few examples of non-rectangular channels include various channels of constructions, panels of electronic equipment and solar energy collectors etc. Several geometrical configurations, more or less complex, have been examined under theoretical, numerical or experimental approaches.

Rahman et al. [1] studied numerically the effect of Prandtl number on hydromagnetic mixed convection in a double-lid driven cavity with a heat-generating obstacle the opposing mixed convection in a vented enclosure. They found that the flow and temperature field were strongly depend on the above stated parameter for the values considered. The variation of the average Nusselt number, the average temperature of the fluid and the temperature at the body centre for various Richardson number had been presented. A numerical analysis of laminar mixed convection in an open cavity with a heated wall bounded by a horizontally insulated plate was presented by Manca et al. [2], where three heating modes were considered: assisting flow, opposing flow and heating from below. Results were reported for Richardson number from 0.1 to 100, Reynolds number from 100 to 1000 and aspect ratio in the range 0.1–1.5. They showed that the maximum temperature values decreased as the Reynolds and the Richardson numbers increased. The effect of the ratio of channel height to the cavity height was found to play a significant role on streamline and isotherm patterns for different heating configurations. The investigation also indicated that opposing forced flow configuration had the highest thermal performance, in terms of both maximum temperature and average Nusselt number. Later, similar problem for the case of assisting forced flow configuration was tested experimentally by Manca et al. [3] and based on the flow visualization results. They pointed out that for *Re* = 1000 there were two nearly distinct fluid motions: a parallel forced flow in the channel and a recirculation flow inside the cavity and for *Re* = 100, the effect of a stronger buoyancy determined a penetration of thermal plume from the heated plate wall into the upper channel.

Unsteady mixed convection in a horizontal channel containing heated blocks on its lower wall was studied numerically by Najam et al. [4]. Tsay et al. [5] rigorously investigated the thermal and hydrodynamic interactions among the surface-mounted heated blocks and baffles in a duct flow mixed convection. They focused particularly the effects of the height of baffle, distance between the heated blocks, baffle and number of baffles on the flow structure and heat transfer characteristics for the system at various *Re* and Gr/Re^2. Bhoite et al. [6] studied numerically the problem of mixed convection flow and heat transfer in a shallow enclosure with a series of block-like heat generating component for a range of Reynolds and Grashof numbers and block-to-fluid thermal conductivity ratios. They showed that higher Reynolds number tend to create a recirculation region of increasing strength at the core region and the effect of buoyancy becomes insignificant beyond a Reynolds number of typically 600 and the thermal conductivity ratio had a negligible effect on the velocity fields.

Brown and Lai [7] numerically studied a horizontal channel with an open cavity and obtained correlations for combined heat and mass transfer which covered the entire convection regime from natural, mixed to forced convection. Saha et. al [8] performed natural convection heat transfer within octagonal enclosure. Their results showed that the effect of *Ra* on the convection heat transfer phenomenon inside the enclosure was significant for all values of *Pr* studied (0.71-50). It was also found that, *Pr* influence natural convection inside the enclosure at high *Ra* ($Ra > 10^4$). Very recently Rahman et al. [9] developed the magnetic field effect on mixed convective flow in a horizontal channel with a bottom heated open enclosure. Their results indicated that the magnetic field strongly affected the flow phenomenon and temperature field inside the cavity whereas this effect was less significant in the channel.

On the basis of the literature review, it appears that no work was reported on mixed convection in an octagonal channel with heat generating circular block. The present study addresses the effects of Prandtl number and Richardson number on the thermal and flow fields for such geometry. The numerical computation covers a wide range of Prandtl number ($0.73 \le Pr \le 7$) and Richardson number ($0.1 \le Ri \le 10$).

2. MODEL SPECIFICATION

The geometry of the problem herein investigated is depicted in Fig. 1. The system consists of an octagonal channel with sides of length *L*, within which a heat generating hollow circular body with outer diameter *d* is centered. The hollow circular body has a thermal conductivity of k_s and generates uniform heat *q* per unit volume. All solid walls of the octagon are considered to be adiabatic. It is assumed that the incoming flow has a uniform velocity, v_i and temperature, T_i. The inlet opening is located at the bottom, whereas the outlet opening is at the top of the octagon.

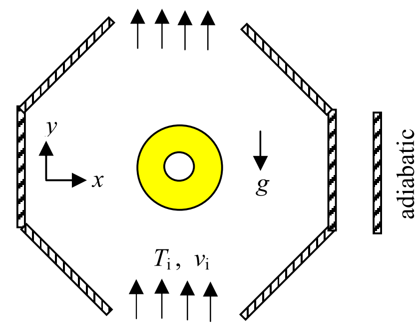


Fig 1**.** Physical model of the vertical channel

3. MATHEMATICAL FORMULATION

A two-dimensional, steady, laminar, incompressible, mixed convection flow is considered within the channel and the fluid properties are assumed to be constant. The dimensionless equations describing the flow under Boussinesq approximation are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

For hollow obstacle the energy equation is

$$\frac{K}{Re\,Pr}\left(\frac{\partial^2 \theta_s}{\partial X^2}+\frac{\partial^2 \theta_s}{\partial Y^2}\right)+Q=0 \tag{5}$$

Here $Re=\frac{v_i L}{\nu}$, $Pr=\frac{\nu}{\alpha}$, $Ri=\frac{g\beta\Delta TL}{v_i^2}$ and $Q=\frac{qL^2}{k_s\Delta T}$ are Reynolds number, Prandtl number, Richardson number and heat generating parameter respectively.

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ D=\frac{d}{L},\ U=\frac{u}{v_i},\ V=\frac{v}{v_i},\ P=\frac{p}{\rho v_i^2},$$

$$\theta=\frac{(T-T_i)}{(T_h-T_i)},\ \theta_s=\frac{(T_s-T_i)}{(T_h-T_i)}$$

The boundary conditions for the present problem are specified as follows:

at the inlet: $U = 0$, $V = 1$, $\theta = 0$

at the outlet: convective boundary condition $P = 0$

at all solid boundaries: $U = 0, V = 0$

at the walls of the octagon: $\frac{\partial \theta}{\partial N}=0$

at the fluid-solid interface: $\left(\frac{\partial \theta}{\partial N}\right)_{fluid} = K\left(\frac{\partial \theta_s}{\partial N}\right)_{solid}$

where N is the dimensional distances either X or Y direction and K is the ratio of solid-fluid thermal conductivity $\left(k_s / k_f\right)$.

The average Nusselt number at the heat generating body is expressed as $Nu = -\frac{1}{L_s}\int_0^{L_s} \frac{\partial \theta}{\partial n}\ dS$,

where $\frac{\partial \theta}{\partial n} = \sqrt{\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2}$. L_s and S are the non-dimensional length and coordinate along the circular surface respectively.

4. COMPUTATIONAL PROCEDURE

The momentum and energy balance equations are the combinations of mixed elliptic-parabolic system of partial differential equations that have been solved by using the Galerkin weighted residual finite element technique. The six node triangular element is used in this work for the development of the finite element equations. All six nodes are associated with velocities as well as temperature. Only three corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation. Firstly, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin's method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using reduced integration technique [10, 11] and Newton-Raphson method [12]. Finally, these linear equations are solved by applying Triangular Factorization method.

4.1 Grid Refinement Check

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for Pr = 1.73, Re = 50, Ri = 1, Q = 5, K = 5 and D = 0.3. The extreme values of Nu and θ_{max} are used as a sensitivity measure of the accuracy of the solution and are selected as the monitoring variables. Considering both the accuracy of numerical values and computational time, the present calculations are performed with 40295 nodes and 10936 elements grid system.

Table 1: Grid Sensitivity Check at Pr = 1.73, Re = 50, Ri = 1, Q = 5, K = 5 and D = 0.3

Nodes (elements)	7224 (4816)	12982 (5784)	26538 (8992)	40295 (10936)	80524 (18080)
Nu	5.31275	5.52682	5.61574	5.71674	5.71674
$\boldsymbol{\theta_{max}}$	1.13283	1.09182	1.07983	1.05182	1.05182
Time (s)	226.265	292.594	388.157	421.328	627.375

4.2 Code Validation

The model comparison is an essential part of a mathematical investigation. Hence, the outcome of the present numerical code is benchmarked graphically against the numerical result of Saha et al. [8] which was reported for natural convection flow and heat transfer within octagonal enclosure. The comparison is conducted while employing the dimensionless parameters Ra = 10^4 and Pr = 0.71. Present result for both the isotherms and streamlines is shown in Fig. 2, which is an outstanding agreement with those of Saha et al. [8]. This justification boosts the assurance in this numerical code to carry on with the above stated objective of the existing investigation.

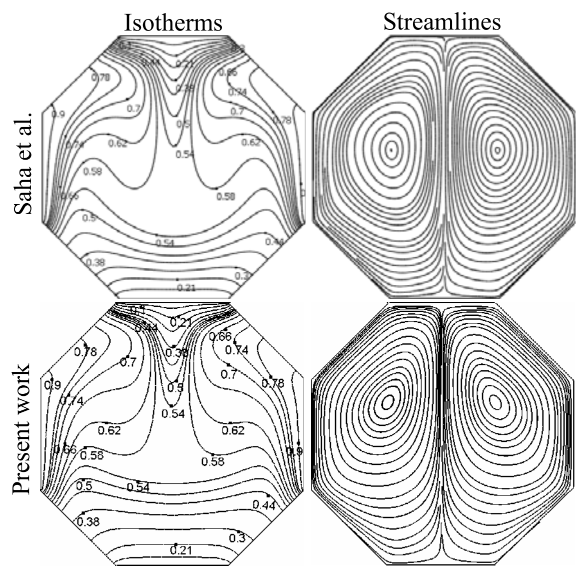
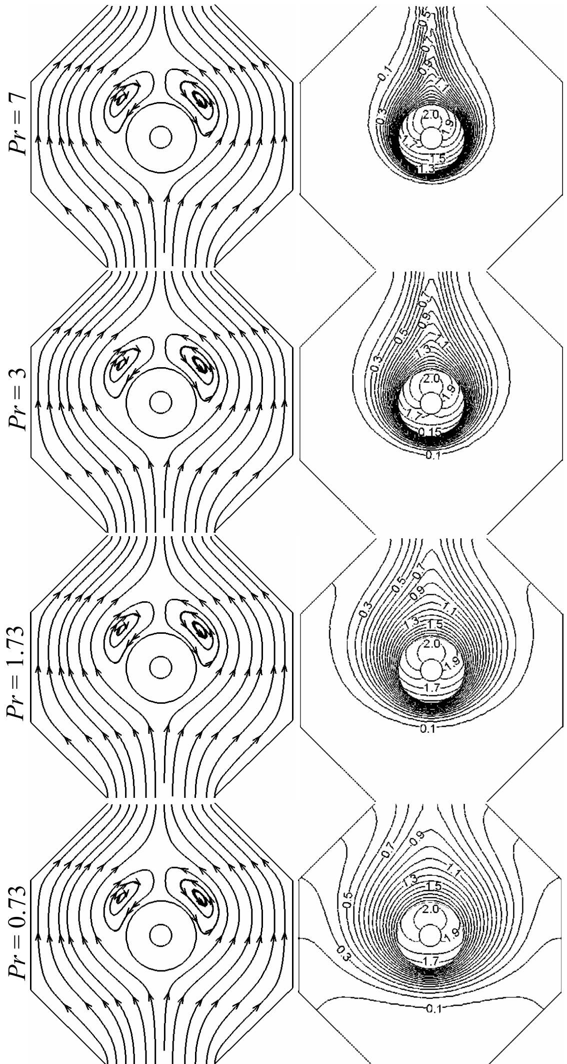


Fig 2. Comparison between present code and Saha et al. [8]

5. RESULTS AND DISCUSSION

The mixed convection phenomenon inside a two sided open octagon having a heat-generating hollow circular block is influenced by different controlling parameters such as Reynolds number Re, Richardson number Ri, Prandtl number Pr, solid fluid thermal conductivity ratio K, heat generating parameter Q and diameter D of the circular body. Analysis of the results is made through obtained streamlines, isotherms, average Nusselt number and maximum temperature of the fluid for three significant parameters, Re, Pr and Ri. The ranges are varied as $0.73 \le Pr \le 7$ and $0.1 \le Ri \le 10$, while the other parameters K, Re, D and Q are kept fixed at 5, 50, 0.3 and 5 respectively.

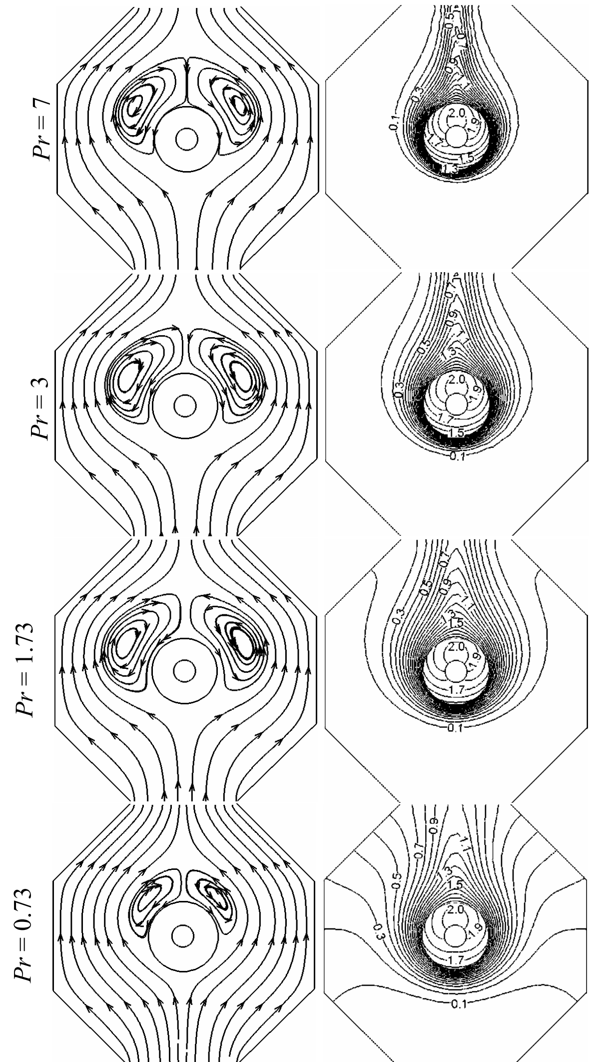


(a) (b)

Fig 3. (a) Streamlines and (b) Isotherms for various Pr at Ri = 0.1 with Re = 50, K =5, D = 0.3 and Q = 5

Fig. 3 (a) – (b) provide the information about the influence of Pr at the forced convection dominated region (Ri = 0.1) on streamlines as well as isotherms. The flow with small Prandtl number (Pr = 0.73) creates a couple of tiny recirculation regions near the heat generating hollow cylinder as shown in Fig. 3 (a). These recirculation regions remain almost identical and the flow structure is not affected by varying Pr. Thermal boundary layer thickness reduces as Pr increases and the isothermal lines become denser at the adjacent area of the heat source. Thus empty space is seen at the bottom, left and right part of the octagon.

The influence of Pr at the mixed convection region (Ri = 1) on flow and temperature fields is depicted in Fig. 4 (a) – (b). The flow pattern changes radically due to variation of Pr. For escalating values of Pr, the eddy present in the flow field rises by size. Consequently, the isothermal lines disappear from the bottom part of the octagon and take an onion shape from the circular body to exit port. This is due to confining the thermal boundary layer in a small region for highly viscous fluid.



(a) (b)

Fig 4**.** (a) Streamlines and (b) Isotherms for various Pr at Ri = 1 with Re = 50, K =5, D = 0.3 and Q = 5

Streamlines and thermal lines are offered in Fig. 5 (a) – (b) with different Prandtl number and purely free convection effect (Ri = 10). The flow field bifurcates near the circular body. Size of spinning cells becomes more large due to buoyancy force with increasing Ri (= 10). The bend in isothermal lines appears due to the high convective current inside the channel. The lines become more concentrated from the forced convection dominant region to free convection dominant regime for a particular Pr.

The variation of the average Nusselt number (Nu) at the heated surface and the maximum temperature of the fluid (θ_{max}) for different Prandtl numbers with Richardson numbers has been presented in Fig. 6 (i) – (ii). It is clearly seen from the figure that for a particular value of Ri, the average Nusselt number is the highest and maximum temperature of fluid is the lowest for the largest Prandtl number Pr = 7. This is because, the fluid with the highest Pr is capable to carry more heat away from the heat source and dissipated through the out flow opening. On the other hand, Nu remains almost constant for lower values of Prandtl number. It decreases slightly for the highest Pr in the forced convection dominated region ($Ri \leq 1$) and beyond this region, it becomes flatten. Moreover, for the lowest value of Pr, θ_{max} grows up sharply upto Ri = 1 and then it is invariant with rising Ri. In addition, for the remaining Pr, it remains constant with varying Ri.

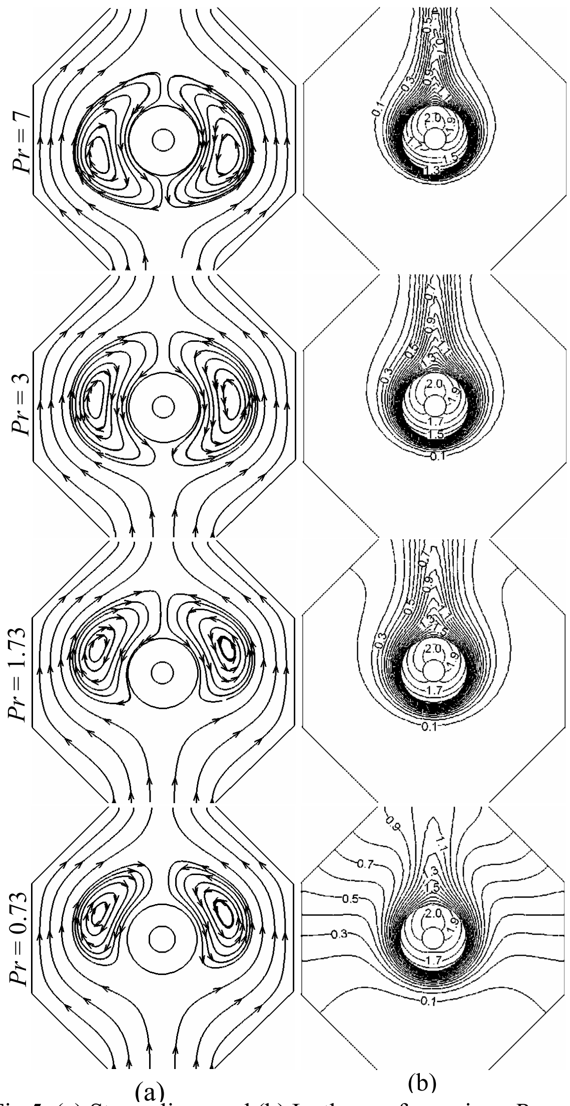


Fig 5. (a) Streamlines and (b) Isotherms for various Pr at Ri = 10 with Re = 50, K =5, D = 0.3 and Q = 5

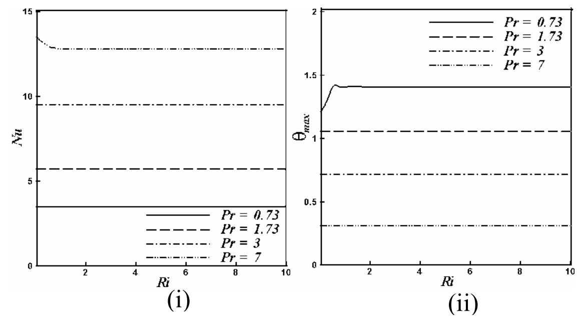


Fig 6. Plots of Pr on (i) average Nusselt number and (ii) maximum temperature of the fluid

6. CONCLUSION

A computational study is performed to investigate the mixed convection in an octagonal channel with a heat-generating horizontal circular body. Results are obtained for wide ranges of Reynolds number Re, Richardson number Ri and Prandtl number Pr. The following conclusions may be drawn from the present investigations:

- Vortices created by inertia force in the streamlines increase and thermal layer near the heated surface become thin and concentrated with increasing values of Pr.
- The influence of Richardson number Ri on streamlines and isotherms are remarkable for the different values of Pr.
- Escalating the Pr and Ri increase the average Nusselt number at the heated surface and devalues the maximum temperature of the fluid.

ACKNOWLEDGEMENT

The current work is supported by the Department of Mathematics, Bangladesh University of Engineering & Technology.

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8. NOMENCLATURE

Symbol	Meaning	Unit
d	outer diameter of obstacle	(m)
D	dimensionless diameter	
g	gravitational acceleration	(ms^{-2})
k	thermal conductivity	$(Wm^{-1}K^{-1})$
K	thermal conductivity ratio	
L	length of each side of octagon	(m)
Nu	Nusselt number	
p	dimensional pressure	(Nm^{-2})
P	non-dimensional	
Pr	Prandtl number	
q	generated heat per unit volume	(Wm^{-2})
Q	heat generating parameter	
Re	Reynolds number	
Ri	Richardson number	
T	dimensional temperature	(K)
u, v	velocity components	(ms^{-1})
U, V	dimensionless components	
x, y	Cartesian coordinates	(m)
X, Y	non-dimensional coordinates	
	Greek symbols	
α	thermal diffusivity	(m^2s^{-1})
β	thermal expansion coefficient	(K^{-1})
υ	kinematic viscosity of the fluid	(m^2s^{-1})
θ	non-dimensional temperature	
ρ	density of the fluid	(Kgm^{-3})
	Subscripts	
max	maximum	
f	fluid	
h	hot	
i	inlet state	
s	solid	

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